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In this instructional activity students also create and critique inductive and deductive arguments concerning congruency, What does it mean for apples to have chirality? This and other explorations in geometry, such as tiling the plane, boxing a tetrahedron, and investigating Euclid's Characteristic, are included in this resource on Euclidean geometry For Students 10th - 12th. In this Euclidean proof worksheet, students observe models and read given proofs. Students determine conclusions based upon the given information.euclidean geometry grade 11 examplesThis one-page worksheet contains ten Euclidean proof problems.This revision worksheet for CAPS term 1 tests all the skills that should have been learnt in the first term. The worksheet tests exponents, surds, equations including inequalities, completing the square, trinomials, and exponentials. It also tests simultaneous equations skills before looking at nature of roots. Finally revision on linear and quadratic patterns is given. The worksheet covers questions on CPI and various finance situations such as investments and hire. This worksheet covers financial documents such as till slips and household accounts as well as budgets and interest. This worksheet also covers tariff systems and car repayments. Finally there is also a fully worked out memorandum. This worksheet goes through the various probability questions covered in grade 11 mathematical literacy. There are questions that ask you to draw and interpret tree diagrams as well as asking you to draw a contingency table. This grade 11 mathematics worksheet builds on the skills of Euclidean geometry and the theorems learnt in grade 11 such as the tan-chord theorem, alternate segments and so on. It also has a fully worked out memorandum. It is very important to be able to determine the break-even point for a company to know how many products you need to sell before you start to make a profit. 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This grade 11 mathematics worksheet on probability tests students skills and their understanding of mutually exclusive events, dependent and independent events, the product rule, Venn diagrams and tree diagrams.Euclidean Geometry Key Concepts Created Date:. X-sheet 10 Euclidean Geometry Mathematics. X-ample Questions. Arseniy Akopyan. Geometry in Figures. This book is a collection of theorems and problems in classical Euclidean geometry. Download euclidean geometry question paper and memo grade 11 document. On this page you can read or download euclidean geometry question paper and memo grade 11 in PDF format. Geometry in Figures - Moscow Center for Continuous Suggested Post Euclidean Geometry Question Paper And Memo Grade 11 Mathematics June Question Paper Grade 11 Function And Euclidean Geometry Memos Euclidean Geometry Grade 10 Question And Memo Grade 11 And 12 Euclidean Geometry Questions And Memos Pdf Euclidean Geometry Grade 11 June Question Papers And Answers Random Post grade12 note book gase lorato pdf it nots in pdf file for hsc boy to boy sixy physical science school based memos grade 12 limpopo provincial capricorn district grade 11 life sciences research project mde november memo life sciences grade 11 download life sciences grade 10 memo crossword puzzle national and regional growth answer key evs his posno zn notes geography pdf download a level l o paper september memo prentice hall literature version of beowulf. If you don't see any interesting for you, use our search form below: Find.Geometry has been a troublesome section in the curriculum for all of my classes, through each year. One of the authors of the Mind Action Series mathematics textbooks had a workshop that I attended. In this workshophe explained his methods and ideas for teaching geometry. His ideas seemed so logical and obviousyet I had not been using them! It lays out five tiers or levels of geometric thought. A learner would have to master each level before progressing to the next one. Let us have a look at each level:. This is where a learner can learn names of figures and recognises a shape as a whole, e.Euclidean Geometry (Paper 3)We must note that a learner has the ideas of the properties; however, they are in isolation. Informal deduction means a learner may be able to follow a given proof, but they will not be able to write and structure it themselves. The learner should have attained Level 3 b y the time they finish Grade 9. This means that the learner must be able to structure and write up formal proofs in the Statement-Reason format. The learner should have attained Level 4 by the time they finish Grade A learner could challenge axioms in different systems and determine if they would still be valid or not, since in the system in which the axioms are used, these axioms may break down. An example of this would be the axiom about the sum of angles in a triangle equalling degrees. This holds if we are looking at normal geometry planar ; however, it does not hold if we are using a rounded surface spherical. This level is for post-matric courses generally. It is important to see the order of these levels since they will clearly demonstrate the problem with how Euclidean Geometry is taught in most textbooks and classrooms. Have you seen the problem yet? It is given before the examples that help you understand how the theorem works. The idea of what a theorem was and how to use it was lost! In my opinion, this is the most important step! It is the step that will help a learner SEE the geometry since they know visually what they are looking for. If the learner does not know what it could look like when the theorem is applicable, how on Earth are they going to be able to see when to use it?!! This is where the learner will learn about properties, i. Since the learner has already learnt the diagram, the statement and reason that they will use makes more sense since the statement refers to what happens in the diagram! This will mean that the linking in the brain of the learner will be easier and therefore, remembering it will be easier. There are many ways that can be used to remember the statements of the theorems. Below, there is an example of using a song to remember them:. This is where another crucial difference in this method appears!Video: An axiom is an established or accepted principle. For this section, the following are accepted as axioms. The theorem of Pythagoras states that the square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides. A theorem is a hypothesis proposition that can be shown to be true by accepted mathematical operations and arguments. A proof is the process of showing a theorem to be correct. The converse of a theorem is the reverse of the hypothesis and the conclusion. If a line is drawn from the centre of a circle perpendicular to a chord, then it bisects the chord. If a line is drawn from the centre of a circle to the mid-point of a chord, then the line is perpendicular to the chord. If the perpendicular bisector of a chord is drawn, then the line will pass through the centre of the circle. Sign up to get a head start on bursary and career opportunities. Use Siyavula Practice to get the best marks possible.Free Euclidean Geometry WorksheetsIf an arc subtends an angle at the centre of a circle and at the circumference, then the angle at the centre is twice the size of the angle at the circumference. If the angles subtended by a chord of the circle are on the same side of the chord, then the angles are equal. From the theorem above we can deduce that if angles at the circumference of a circle are subtended by arcs of equal length, then the angles are equal. In the figure below, notice that if we were to move the two chords with equal length closer to each other, until they overlap, we would have the same situation as with the theorem above.euclidean geometry pdf grade 11This shows that the angles subtended by arcs of equal length are also equal. If a line segment subtends equal angles at two other points on the same side of the line segment, then these four points are concyclic lie on a circle. Points on the circumference of a circle: we know that there are only two possible options regarding a given point — it either lies on circumference or it does not. Cyclic quadrilaterals are quadrilaterals with all four vertices lying on the circumference of a circle concyclic. If the interior opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic. A tangent is a line that touches the circumference of a circle at only one place. The radius of a circle is perpendicular to the tangent at the point of contact. The angle between a tangent to a circle and a chord drawn at the point of contact, is equal to the angle which the chord subtends in the alternate segment. If a line drawn through the end point of a chord forms an angle equal to the angle subtended by the chord in the alternate segment, then the line is a tangent to the circle. Motivate your answer.Rick warren podcast archivePerpendicular line from circle centre bisects chord If a line is drawn from the centre of a circle perpendicular to a chord, then it bisects the chord. Theorem: Perpendicular bisector of chord passes through circle centre If the perpendicular bisector of a chord is drawn, then the line will pass through the centre of the circle. Success in Maths and Science unlocks opportunities Sign up to get a head start on bursary and career opportunities. Sign up to unlock your future. Exercise 8. Theorem: Angle at the centre of a circle is twice the size of the angle at the circumference If an arc subtends an angle at the centre of a circle and at the circumference, then the angle at the centre is twice the size of the angle at the circumference. Use theorems and the given information to find all equal angles and sides on the diagram.Financial accounting chapter 9Conclusion The diameter of a circle subtends a right angle at the circumference angles in a semi-circle. Theorem: Subtended angles in the same segment of a circle are equal If the angles subtended by a chord of the circle are on the same side of the chord, then the angles are equal. Proof by contradiction: Points on the circumference of a circle: we know that there are only two possible options regarding a given point — it either lies on circumference or it does not.Video: An axiom is an established or accepted principle. For this section, the following are accepted as axioms. The theorem of Pythagoras states that the square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides. A tangent is perpendicular to the radiusdrawn at the point of contact with the circle. A theorem is a hypothesis proposition that can be shown to be true by accepted mathematical operations and arguments. A proof is the process of showing a theorem to be correct. The converse of a theorem is the reverse of the hypothesis and the conclusion. If a line is drawn from the centre of a circle perpendicular to a chord, then it bisects the chord. Reason: from centre bisects chord. Circle with centre and line perpendicular to chord. Draw and. In and inand Therefore bisects. In and inTherefore bisects. If a line is drawn from the centre of a circle to the mid-point of a chord, then the line is perpendicular to the chord. Reason: line from centre to mid-point. Circle with centre and line to mid-point on chord. In and inTherefore. If the perpendicular bisector of a chord is drawn, then the line will pass through the centre of the circle. Reason: bisector through centre. Circle with mid-point on chord. Line is drawn such that. Draw lines and. In and in. Similarly it can be shown that in and in. We conclude that all the points that are equidistant from and will lie on the line extended. Therefore the centrewhich is equidistant to all points on the circumference, must also lie on the line.To browse Academia. Skip to main content. Log In Sign Up. So, be sure to study the vocabulary as well as the facts which you gather. NOTE: The plural of vertex is vertices!Minio adminGiven : reflex AOD e. Is ADB a straight line? Give a reason for your answer. Their respective sides will also be in proportion. The exterior angle of a triangle equals the sum 2 of the interior opposite angles. However, we Answers. Only 1 option is possible.Bypass proxyThe exterior angle of a triangle equals the SUM of the two interior opposite angles. The side restricts the size of the triangle. However, Why is the exterior angle of a triangle equal to when comparing triangles, the given equal sides must correspond the sum of the two interior opposite angles? If 2 triangles are equal in every respect i. It will have the same shape all angles will be C the same size as before but the respective sides will not be the same length.B A They will be proportional. This package is an extract from our Gr 10 Maths 3 in 1 study guide. B C We trust that this will help you to grow in confidence as you prepare for your exams. Visit our website to find appropriate resources for your success! Euclidean geometry grade 11 questions and answers pdf Terminology The following terms are regularly used when referring to circles: Arc — a portion of the circumference of a circle. Chord — a straight line joining the ends of an arc. Circumference — the perimeter or boundary line of a circle. Radius (\(r\)) — any straight line from the centre of the circle to a point on the circumference. Diameter — a special chord that passes through the centre of the circle. A diameter is a straight line segment from one point on the circumference to another point on the circumference that passes through the centre of the circle. Segment — part of the circle that is cut off by a chord. A chord divides a circle into two segments. Tangent — a straight line that makes contact with a circle at only one point on the circumference. Axioms An axiom is an established or accepted principle. For this section, the following are accepted as axioms. The theorem of Pythagoras states that the square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides. \[(AC)^2 = (AB)^2 + (BC)^2\] A tangent is perpendicular to the radius (\(OT \perp ST\)), drawn at the point of contact with the circle. A theorem is a hypothesis (proposition) that can be shown to be true by accepted mathematical operations and arguments. A proof is the process of showing a theorem to be correct. The converse of a theorem is the reverse of the hypothesis and the conclusion. For example, given the theorem "if \(A\), then \(B\)", the converse is "if \(B\), then \(A\)". If a line is drawn from the centre of a circle perpendicular to a chord, then it bisects the chord. (Reason: \(\perp\) from centre bisects chord) Circle with centre \(O\) and line \(OP\) perpendicular to chord \(AB\). Draw \(OA\) and \(OB\). In \(\triangle OPA\) and in \(\triangle OPB\), \[\begin{array}{rll} OA^2 &= OP^2 + AP^2 & \text{(Pythagoras)} \\ OB^2 &= OP^2 + BP^2 & \text{(Pythagoras)} \end{array}\] and \[\begin{array}{rll} OA &= OB & \text{(equal radii)} \\ \therefore AP^2 &= BP^2 & \\ \therefore AP &= BP & \end{array}\] Therefore \(OP\) bisects \(AB\). Alternative proof: In \(\triangle OPA\) and in \(\triangle OPB\), \[\begin{array}{rll} O\hat{P}A &= O\hat{P}B & (\text{given } OP \perp AB) \\ OA &= OB & \text{(equal radii)} \\ OP &= OP & \text{(common side)} \\ \therefore \triangle OPA & \equiv \triangle OPB & \text{(RHS)} \\ \therefore AP &= PB & \end{array}\] Therefore \(OP\) bisects \(AB\). If a line is drawn from the centre of a circle to the mid-point of a chord, then the line is perpendicular to the chord. (Reason: line from centre to mid-point \(\perp\)) Circle with centre \(O\) and line \(OP\) to mid-point \(P\) on chord \(AB\). Draw \(OA\) and \(OB\). In \(\triangle OPA\) and in \(\triangle OPB\), \[\begin{array}{rll} OA &= OB & \text{(equal radii)} \\ AP &= PB & \text{(given)} \\ OP &= OP & \text{(common side)} \\ \therefore \triangle OPA & \equiv \triangle OPB & \text{(SSS)} \\ \therefore O\hat{P}A &= O\hat{P}B & \\ \text{and } O\hat{P}A + O\hat{P}B &= \text{180}\text{°} & (\angle \text{ on str. line}) \\ \therefore O\hat{P}A = O\hat{P}B &= \text{90}\text{°} & \end{array}\] Therefore \(OP \perp AB\). If the perpendicular bisector of a chord is drawn, then the line will pass through the centre of the circle. (Reason: \(\perp\) bisector through centre) Circle with mid-point \(P\) on chord \(AB\). Line \(QP\) is drawn such that \(Q\hat{P}A = Q\hat{P}B = \text{90}\text{°}\). Line \(RP\) is drawn such that \(R\hat{P}A = R\hat{P}B = \text{90}\text{°}\). Circle centre \(O\) lies on the line \(PR\) Draw lines \(QA\) and \(QB\). Draw lines \(RA\) and \(RB\). In \(\triangle QPA\) and in \(\triangle QPB\), \[\begin{array}{rll} AP &= PB & \text{(given)} \\ QP &= QP & \text{(common side)} \\ Q\hat{P}A = Q\hat{P}B &= \text{90}\text{°} & \text{(given)} \\ \therefore \triangle QPA & \equiv \triangle QPB & \text{(SAS)} \\ \therefore QA &= QB & \end{array}\] Similarly it can be shown that in \(\triangle RPA\) and in \(\triangle RPB\), \(RA = RB\). We conclude that all the points that are equidistant from \(A\) and \(B\) will lie on the line \(PR\) extended. Therefore the centre \(O\), which is equidistant to all points on the circumference, must also lie on the line \(PR\). Given \(OQ \perp PR\) and \(PR = 8\) units, determine the value of \(x\). \(PQ = QR = 4 \qquad (\perp \text{ from centre bisects chord})\) In \(\triangle OQP\): \[\begin{array}{rll} PQ &= 4 & (\perp \text{ from centre bisects chord}) \\ OP^2 &= OQ^2 + QP^2 & (\text{Pythagoras}) \\ 5^2 &= x^2 + 4^2 & \\ \therefore x^2 &= 25 - 16 & \\ x^2 &= 9 & \\ x &= 3 & \end{array}\] Sign up to get a head start on bursary and career opportunities. Use Siyavula Practice to get the best marks possible.Sign up to unlock your futureExercise 8.1 In the circle with centre \(O\), \(OQ \perp PR\), \(OQ = 4\) units and \(PR=10\). Determine \(x\). \[\begin{array}{rll} PR &= 10 & (\perp \text{ given }) \\ \therefore PQ &= 5 & (\perp \text{ from centre bisects chord}) \\ OP^2 &= OQ^2 + QP^2 & (\text{Pythagoras}) \\ x^2 &= 4^2 + 5^2 & \\ \therefore x^2 &= 25 + 16 & \\ x^2 &= 41 & \\ x &= \sqrt{41} & \end{array}\] In the circle with centre \(O\) and radius \(= 10\) units, \(OQ \perp PR\) and \(PR=8\). Determine \(x\). \[\begin{array}{rll} PR &= 8 & (\perp \text{ given }) \\ \therefore PQ &= 4 & (\perp \text{ from centre bisects chord}) \\ OP^2 &= OQ^2 + QP^2 & (\text{Pythagoras}) \\ 10^2 &= x^2 + 4^2 & \\ \therefore x^2 &= 100 - 16 & \\ x^2 &= 84 & \\ x &= \sqrt{84} & \end{array}\] In the circle with centre \(O\), \(OQ \perp PR\), \(PR = 12\) units and \(SQ = 2\) units. Determine \(x\). \[\begin{array}{rll} SO &= x -2 & \\ OP^2 &= OS^2 + SP^2 & (\text{Pythagoras}) \\ x^2 &= (x-2)^2 + 6^2 & \\ x^2 &= x^2 - 4x + 4 + 6^2 & \\ 4x &= 40 & \\ \therefore x &= 10 & \end{array}\] In the circle with centre \(O\), \(OT \perp SQ\), \(OT \perp PR\), \(OP = 10\) units, \(ST = 5\) units and \(PU = 8\) units. Determine \(TU\). \[\begin{array}{rll} \text{In } \triangle POU, \quad OP^2 &= OU^2 + PU^2 & (\text{Pythagoras}) \\ 10^2 &= OU^2 + 8^2 & \\ 100 - 64 &= OU^2 & \\ \therefore OU^2 &= 36 & \\ \therefore OU &= 6 & \\ \text{In } \triangle QTO, \quad QO^2 &= OT^2 + TQ^2 & (\text{Pythagoras}) \\ 10^2 &= OT^2 + 5^2 & \\ 100 - 25 &= OT^2 & \\ \therefore OT^2 &= 75 & \\ \therefore OT &= \sqrt{75} & \\ \therefore TU &= OT - OU & \\ &= \sqrt{75} - 6 & \\ &= \text{2,66} & \end{array}\] In the circle with centre \(O\), \(OT \perp QP\), \(OS \perp PR\), \(OT = 5\) units, \(PQ = 24\) units and \(PR = 25\) units. Determine \(OS = x\). \[\begin{array}{rll} \text{In } \triangle QTO, \quad QO^2 &= OT^2 + QT^2 & (\text{Pythagoras}) \\ QO^2 &= 5^2 + 12^2 & \\ &= 25 + 144 & \\ \therefore QO^2 &= 169 & \\ \therefore QO &= 13 & \\ \text{In } \triangle OSR, \quad OR^2 &= SR^2 + OS^2 & (\text{Pythagoras}) \\ 13^2 &= \text{12,5}^2 + OS^2 & \\ \therefore OS^2 &= \text{12,75} & \\ \therefore OS &= \text{3,6} & \end{array}\] Measure angles \(x\) and \(y\) in each of the following graphs: Complete the table: Use your results to make a conjecture about the relationship between angles subtended by an arc at the centre of a circle and angles at the circumference of a circle. Now draw three of your own similar diagrams and measure the angles to check your conjecture. If an arc subtends an angle at the centre of a circle and at the circumference, then the angle at the centre is twice the size of the angle at the circumference. (Reason: \(\angle \text{ at centre } = 2 \angle \text{ at circum.}\)) Circle with centre \(O\), arc \(AB\) subtending \(A\hat{O}B\) at the centre of the circle, and \(A\hat{P}B\) at the circumference. \(A\hat{O}B= 2A\hat{P}B\) Draw \(PO\) extended to \(Q\) and let \(A\hat{O}Q = \hat{O}_1\) and \(B\hat{O}Q = \hat{O}_2\). \[\begin{array}{rll} \hat{O}_1&= A\hat{P}O + P\hat{A}O & (\text{ext. } \angle \triangle = \text{sum int. opp. } \angle \text{s}) \\ \text{and } A\hat{P}O &= P\hat{A}O & (\text{equal radii, isosceles } \triangle APO)\\ \therefore \hat{O}_1&= A\hat{P}O + A\hat{P}O & \\ \hat{O}_1&= 2A\hat{P}O & \end{array}\] Similarly, we can also show that \(\hat{O}_2 = 2B\hat{P}O\). For the first two diagrams shown above we have that: \[\begin{array}{rll} A\hat{O}B &= \hat{O}_1 + \hat{O}_2 & \\ &= 2A\hat{P}O + 2B\hat{P}O & \\ &= 2(A\hat{P}O + B\hat{P}O) & \\ \therefore A\hat{O}B &= 2(A\hat{P}B) & \end{array}\] And for the last diagram: \[\begin{array}{rll} A\hat{O}B &= \hat{O}_2 - \hat{O}_1 & \\ &= 2B\hat{P}O - 2A\hat{P}O & \\ &= 2(B\hat{P}O - A\hat{P}O) & \\ \therefore A\hat{O}B &= 2(A\hat{P}B) & \end{array}\] Given \(HK\), the diameter of the circle passing through centre \(O\). In \(\triangle HJK\): \[\begin{array}{rll} H\hat{O}K &= \text{180}\text{°} & (\angle \text{ on str. line}) \\ &= 2a & (\angle \text{ at centre } = 2 \angle \text{ at circum.}) \\ \therefore 2a &= \text{180}\text{°} & \\ a &= \frac{\text{180}\text{°}}{2} & \\ &= \text{90}\text{°} & \end{array}\] The diameter of a circle subtends a right angle at the circumference (angles in a semi-circle). Sign up to get a head start on bursary and career opportunities. Use Siyavula Practice to get the best marks possible.Sign up to unlock your futureExercise 8.2 \[\begin{array}{rll} b &= 2 \times \text{45}\text{°} & (\angle \text{ at centre } = 2 \angle \text{ at circum.}) \\ \therefore b &= \text{90}\text{°} & \end{array}\] \[\begin{array}{rll} c &= \frac{1}{2} \times \text{45}\text{°} & (\angle \text{ at centre } = 2 \angle \text{ at circum.}) \\ \therefore c &= \text{22,5}\text{°} & \end{array}\] \[\begin{array}{rll} d &= 2 \times \text{100}\text{°} & (\angle \text{ at centre } = 2 \angle \text{ at circum.}) \\ \therefore d &= \text{200}\text{°} & \end{array}\] \[\begin{array}{rll} e &= \text{100}\text{°} - \text{90}\text{°} - \text{35}\text{°} & (\angle \text{ in semi circle }) \\ \therefore e &= \text{55}\text{°} & \end{array}\] \[\begin{array}{rll} f &= \frac{1}{2} \times \text{240}\text{°} & (\angle \text{ at centre } = 2 \angle \text{ at circum.}) \\ \therefore f &= \text{120}\text{°} & \end{array}\] Measure angles \(a\), \(b\), \(c\), \(d\) and \(e\) in the diagram below: Choose any two points on the circumference of the circle and label them \(A\) and \(B\). Draw \(AP\) and \(BP\), and measure \(A\hat{P}B\). Draw \(AQ\) and \(BQ\), and measure \(A\hat{Q}B\). What do you observe? Make a conjecture about these types of angles. If the angles subtended by a chord of the circle are on the same side of the chord, then the angles are equal. (Reason: \(\angle\)s in same seg.) Circle with centre \(O\), and points \(P\) and \(Q\) on the circumference of the circle. Arc \(AB\) subtends \(A\hat{P}B\) and \(A\hat{Q}B\) in the same segment of the circle. \(A\hat{P}B = A\hat{Q}B\) \[\begin{array}{rll} A\hat{O}B &= 2 A\hat{P}B & (\angle \text{ at centre } = 2 \angle \text{ at circum.}) \\ A\hat{O}B &= 2 A\hat{Q}B & (\angle \text{ at centre } = 2 \angle \text{ at circum.}) \\ \therefore 2 A\hat{P}B &= 2 A\hat{Q}B & \\ A\hat{P}B &= A\hat{Q}B & \end{array}\] Equal arcs subtend equal angles From the theorem above we can deduce that if angles at the circumference of a circle are subtended by arcs of equal length, then the angles are equal. In the figure below, notice that if we were to move the two chords with equal length closer to each other, until they overlap, we would have the same situation as with the theorem above. This shows that the angles subtended by arcs of equal length are also equal. If a line segment subtends equal angles at two other points on the same side of the line segment, then these four points are concyclic (lie on a circle). Line segment \(AB\) subtending equal angles at points \(P\) and \(Q\) on the same side of the line segment \(AB\). \(A\), \(B\), \(P\) and \(Q\) lie on a circle. Proof by contradiction: Points on the circumference of a circle: we know that there are only two possible options regarding a given point — it either lies on circumference or it does not. We will assume that point \(P\) does not lie on the circumference. We draw a circle that cuts \(AP\) at \(R\) and passes through \(A\), \(B\) and \(Q\). \[\begin{array}{rll} A\hat{Q}B &= A\hat{R}B & (\angle \text{s in same seg.}) \\ \text{but } A\hat{Q}B &= A\hat{P}B & (\text{given}) \\ \therefore A\hat{R}B &= A\hat{P}B & \\ \text{but } A\hat{R}B &= A\hat{P}B + R\hat{B}P & (\text{ext. } \angle \triangle = \text{sum int. opp.}) \\ \therefore R\hat{B}P &= \text{0}\text{°} & \end{array}\] Therefore the assumption that the circle does not pass through \(P\) must be false. We can conclude that \(A\), \(B\), \(Q\) and \(P\) lie on a circle (\(A\), \(B\), \(Q\) and \(P\) are concyclic). Given \(FH \parallel EI\) and \(E\hat{I}F = \text{15}\text{°}\), determine the value of \(b\). \[\begin{array}{rll} H\hat{F}I &= \text{15}\text{°} & (\text{alt. } \angle, FH \parallel EI) \\ \text{and } b &= H\hat{F}I & (\angle \text{s in same seg.}) \\ \therefore b &= \text{15}\text{°} & \end{array}\] Exercise 8.3 \[\begin{array}{rll} a &= \text{21}\text{°} & (\angle \text{s in same seg.}) \end{array}\] \[\begin{array}{rll} c &= \text{24}\text{°} & (\angle \text{s in same seg.}) \\ d &= \text{102}\text{°} - \text{24}\text{°} & (\text{ext. } \angle \triangle = \text{sum int. opp.}) \\ \therefore d &= \text{78}\text{°} & \\ \end{array}\] \[\begin{array}{rll} d &= \hat{N} & (\text{alt. } \angle, PO \parallel QN) \\ \hat{N} &= \frac{1}{2} \times \text{17}\text{°} & (\angle \text{ at centre } = 2 \angle \text{ at circum.}) \\ \hat{O} &= \hat{N} & (\angle \text{s in same seg.}) \\ \text{17}\text{°} &= \hat{O} + d & (\text{ ext. angle of } \triangle) \\ \therefore 2d &= \text{17}\text{°} & \\ \therefore d &= \text{8,5}\text{°} & (\text{alt. } \angle, PO \parallel QN) \end{array}\] Given \(T\hat{V}S = S\hat{V}R\), determine the value of \(e\). \[\begin{array}{rll} \text{In } \triangle TRV, \hat{T} &= \text{180}\text{°} - (\text{80}\text{°} + \text{30}\text{°}) & (\angle \text{s sum of } \triangle) \\ \therefore \hat{T} &= \text{70}\text{°} & \\ \therefore e &= \text{15}\text{°} + \text{70}\text{°} & \\ &= \text{85}\text{°} & \\ \end{array}\] Is \(TV\) a diameter of the circle? Explain your answer. No, since \(\text{45}\text{°} + \text{35}\text{°} e \text{90}\text{°}\) Given circle with centre \(O\), \(WT = TY\) and \(X\hat{W}T = \text{35}\text{°}\). Determine \(f\). \[\begin{array}{rll} \text{In } \triangle WTZ &\text{ and in } \triangle YTZ, \\ WT &= YT & (\text{ given }) \\ ZT &= ZT & (\text{ common side }) \\ Y\hat{T}Z = W\hat{T}Z &= \text{90}\text{°} & (\text{ line from circle centre to mid-point }) \\ \therefore T\hat{Z}Y &= T\hat{Z}W & (\text{ SAS }) \\ T\hat{Z}Y = T\hat{Z}W &= f & \\ \text{And } T\hat{Z}Y &= \text{35}\text{°} & (\angle \text{s in same seg.}) \\ \therefore T\hat{Z}W = f &= \text{35}\text{°} & \end{array}\] Cyclic quadrilaterals Cyclic quadrilaterals are quadrilaterals with all four vertices lying on the circumference of a circle (concyclic). Consider the diagrams given below: Circle \(\text{1}\)Circle \(\text{2}\)Circle \(\text{3}\) Complete the following: \(ABCD\) is a cyclic quadrilateral because \(\ldots \ldots\) Complete the table: Circle \(\text{1}\)Circle \(\text{2}\)Circle \(\text{3}\)\(\hat{A} =\)\(\hat{B} =\)\(\hat{C} =\)\(\hat{D} =\)\(\hat{A} + \hat{C} =\)\(\hat{B} + \hat{D} =\) Use your results to make a conjecture about the relationship between angles of cyclic quadrilaterals. The opposite angles of a cyclic quadrilateral are supplementary. (Reason: opp. \(\angle\)s cyclic quad.) Circle with centre \(O\) with points \(A, B, P\) and \(Q\) on the circumference such that \(ABPQ\) is a cyclic quadrilateral. \(A\hat{B}P + A\hat{Q}P = \text{180}\text{°}\) and \(Q\hat{A}B + Q\hat{P}B = \text{180}\text{°}\) Draw \(AO\) and \(OP\). Label \(\hat{O}_1\) and \(\hat{O}_2\). \[\begin{array}{rll} \hat{O}_1 &= 2A\hat{B}P & (\angle\text{ at centre} = 2\angle \text{ at circum.})\\ \hat{O}_2 &= 2A\hat{Q}P & (\angle\text{ at centre} = 2\angle \text{ at circum.})\\ \text{and } \hat{O}_1 + \hat{O}_2 &= \text{360}\text{°} & (\angle\text{s around a point}) \\ \therefore 2A\hat{B}P + 2A\hat{Q}P &= \text{360}\text{°} & \\ A\hat{B}P + A\hat{Q}P &= \text{180}\text{°} & \end{array}\] Similarly, we can show that \(Q\hat{A}B + Q\hat{P}B = \text{180}\text{°}\). Converse: interior opposite angles of a quadrilateral If the interior opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic. Exterior angle of a cyclic quadrilateral If a quadrilateral is cyclic, then the exterior angle is equal to the interior opposite angle. Given the circle with centre \(O\) and cyclic quadrilateral \(PQRS\). \(SQ\) is drawn and \(S\hat{P}Q = \text{34}\text{°}\). Determine the values of \(a\), \(b\) and \(c\). \[\begin{array}{rll} S\hat{P}Q + c &= \text{180}\text{°} & (\text{opp. } \angle \text{s cyclic quad supp.}) \\ \therefore c &= \text{180}\text{°} - \text{34}\text{°} & \\ &= \text{146}\text{°} & \end{array}\] \[\begin{array}{rll} a &= \text{90}\text{°} & (\angle \text{ in semi circle}) \end{array}\] In \(\triangle PSQ\): \[\begin{array}{rll} a + b + \text{34}\text{°} &= \text{180}\text{°} & (\angle \text{ sum of } \triangle) \\ \therefore b &= \text{180}\text{°} - \text{90}\text{°} - \text{34}\text{°} & \\ &= \text{56}\text{°} & \end{array}\] Methods for proving a quadrilateral is cyclic There are three ways to prove that a quadrilateral is a cyclic quadrilateral: Method of proofReasonIf \(\hat{P} + \hat{R} = \text{180}\text{°}\) or \(\hat{S} + \hat{Q} = \text{180}\text{°}\), then \(PQRS\) is a cyclic quad.opp. int. angles suppl.If \(\hat{P} = \hat{Q}\) or \(\hat{S} = \hat{R}\), then \(PQRS\) is a cyclic quad.angles in the same seg.If \(T\hat{Q}R = \hat{S}\), then \(PQRS\) is a cyclic quad.ext. angle equal to int. opp. angle Prove that \(ABDE\) is a cyclic quadrilateral. \[\begin{array}{rll} D\hat{B}C &= \text{90}\text{°} & (\angle \text{ in semi circle}) \\ \text{and } \hat{E} &= \text{90}\text{°} & (\text{given}) \\ \therefore D\hat{B}C &= \hat{E} & \\ \therefore ABDE \text{ is a cyclic} & \text{quadrilateral} & \text{(ext.\@ \(\angle\) equals int.\@ opp.\@ \(\angle\))} \end{array}\] Exercise 8.4 \[\begin{array}{rll} a + \text{87}\text{°} &= \text{180}\text{°} & (\text{opp. angles of cyclic quad. supp. }) \\ \therefore a &= \text{93}\text{°} & \\ b + \text{106}\text{°} &= \text{180}\text{°} & (\text{opp. angles of cyclic quad. supp. }) \\ \therefore b &= \text{74}\text{°} & \end{array}\] \[\begin{array}{rll} a &= H\hat{I}J & \angle (\text{ ext. angle cyclic quad = int. opp }) \\ &= \text{114}\text{°} & \end{array}\] \[\begin{array}{rll} \hat{W} + \text{86}\text{°} &= \text{180}\text{°} & (\text{opp. angles of cyclic quad. supp. }) \\ \therefore \hat{W} &= \text{94}\text{°} & \\ a + \hat{W} + \text{57}\text{°} &= \text{180}\text{°} & (\text{angles sum of } \triangle) \\ \therefore a &= \text{29}\text{°} & \end{array}\] \[\begin{array}{rll} A\hat{M}B &= \text{32}\text{°} + D\hat{B}C & (\text{ext angle of } \triangle = \text{sum int. opp. angles}) \\ \therefore \text{72}\text{°} &= \text{32}\text{°} + D\hat{B}C & \\ \therefore D\hat{B}C &= \text{40}\text{°} & (\text{angles sum of } \triangle) \\ \therefore D\hat{B}C &= D\hat{A}C & \\ \text{Therefore } ABCD &= \text{ is cyclic quad. } & (\text{ angles in same seg.}) \end{array}\] \[\begin{array}{rll} \text{In } \triangle ABD, \quad A\hat{B}D = A\hat{D}B &= \text{35}\text{°} & (\angle \text{s opp. equal sides }) \\ \therefore \text{35}\text{°} + \text{35}\text{°} + D\hat{A}B &= \text{180}\text{°} & (\angle \text{s sum of } \triangle) \\ \therefore D\hat{A}B &= \text{110}\text{°} & (\text{angles sum of } \triangle) \\ \text{And } D\hat{A}B + D\hat{C}B &= \text{180}\text{°} & \\ \text{Therefore } ABCD &= \text{ is cyclic quad. } & (\text{ opp. int. angles supp.}) \end{array}\] Tangent line to a circle A tangent is a line that touches the circumference of a circle at only one place. The radius of a circle is perpendicular to the tangent at the point of contact. If two tangents are drawn from the same point outside a circle, then they are equal in length. (Reason: tangents from same point equal) Circle with centre \(O\) and tangents \(PA\) and \(PB\), where \(A\) and \(B\) are the respective points of contact for the two lines. In \(\triangle AOP\) and \(\triangle BOP\), \[\begin{array}{rll} O\hat{A}P = O\hat{B}P &= \text{90}\text{°} & (\text{tangent} \perp \text{radius})\\ AO &= BO & (\text{equal radii}) \\ OP &= OP & (\text{common side}) \\ \therefore \triangle AOP &\equiv \triangle BOP & (\text{RHS}) \\ \therefore AP &= BP & \end{array}\] In the diagram below \(AE = \text{5}\text{ cm}\), \(AC = \text{8}\text{ cm}\) and \(CE = \text{9}\text{ cm}\). Determine the values of \(a\), \(b\) and \(c\). \[\begin{array}{rll} AB = AF&= a & (\text{tangents from } A) \\ EF = ED&= c & (\text{tangents from } E) \\ CB = CD&= b & (\text{tangents from } C) \\ \therefore AE = a + c &= 5 & \\ \text{and } AC = a + b &= 8 & \\ \text{and } CE = b + c &= 9 & \end{array}\] \[\begin{array}{rll} a + c &= 5 & \ldots (1)\\ a + b &= 8 & \ldots (2)\\ b + c &= 9 & \ldots (3) \end{array}\] Subtract equation \((1)\) from equation \((2)\) and then substitute into equation \((3)\): \[\begin{array}{rll} (2) - (1) \quad b-c &= 8-5 & \\ &= 3 & \\ \therefore b &= c + 3 & \\ \text{Substitute into } (3) \quad c + 3 + c &= 9 & \\ 2c &= 6 & \\ c &= 3 & \\ \therefore a &= 2 & \\ \text{ and } b &= 6 & \end{array}\] Exercise 8.5 \[\begin{array}{rll} HI &= HG & (\text{tangents same pt. }) \\ \text{In } \triangle HIJ, \quad d^2 &= 8^2 + 5^2 & (\text{Pythagoras, radius perp. tangent }) \\ \therefore d^2 &= 89 & \\ \therefore d &= \text{9,4}\text{ cm} & \end{array}\] \[\begin{array}{rll} LM = LK &= 6 & (\text{tangents same pt. }) \\ LN &= \text{7,5}\text{ cm} & (\text{ given }) \\ \therefore MN = \text{7,5} - \text{6} &= \text{1,5}\text{ cm} & \\ OM &= \text{2}\text{ cm} & (\text{ radius }) \\ \text{In } \triangle OMN, \quad e^2 &= 2^2 + (\text{1,5})^2 & (\text{Pythagoras, radius perp. tangent }) \\ \therefore e^2 &= \text{6,25} & \\ \therefore e &= \text{2,5}\text{ cm} & \end{array}\] Consider the diagrams given below: Diagram \(\text{1}\)Diagram \(\text{2}\)Diagram \(\text{3}\) Measure the following angles with a protractor and complete the table: Diagram \(\text{1}\)Diagram \(\text{2}\)Diagram \(\text{3}\)\(A\hat{B}C =\)\(\hat{D} =\)\(\hat{E} =\) Use your results to complete the following: the angle between a tangent to a circle and a chord is \(\ldots \ldots\) to the angle in the alternate segment. The angle between a tangent to a circle and a chord drawn at the point of contact, is equal to the angle which the chord subtends in the alternate segment. (Reason: tan. chord theorem) Circle with centre \(O\) and tangent \(SR\) touching the circle at \(B\). Chord \(AB\) subtends \(\hat{P}_1\) and \(\hat{Q}_1\). \(A\hat{B}R = A\hat{P}B\) \(A\hat{B}S = A\hat{Q}B\) Draw diameter \(BT\) and join \(T\) to \(A\). Let \(A\hat{T}B = T_1\). \[\begin{array}{rll} A\hat{B}S + A\hat{B}T &= \text{90}\text{°} & (\text{tangent} \perp \text{radius}) \\ B\hat{A}T &= \text{90}\text{°} & (\angle \text{ in semi circle}) \\ \therefore A\hat{B}T + T_1 &= \text{90}\text{°} & (\angle \text{ sum of } \triangle BAT) \\ \therefore A\hat{B}S &= T_1 & \\ \text{but } Q_1 &= T_1 & (\angle \text{s in same segment}) \\ \therefore Q_1 &= A\hat{B}S & \end{array}\] \[\begin{array}{rll} A\hat{B}S + A\hat{B}R &= \text{180}\text{°} & (\angle \text{s on str. line}) \\ \hat{Q}_1 + \hat{P}_1 &= \text{180}\text{°} & (\text{opp. } \angle \text{s cyclic quad. supp.}) \\ \therefore A\hat{B}S + A\hat{B}R &= Q_1 + P_1 & \\ \text{and } A\hat{B}S &= Q_1 & \\ \therefore A\hat{B}R &= P_1 & \end{array}\] Determine the values of \(h\) and \(s\). \[\begin{array}{rll} O\hat{Q}S &= S\hat{R}Q & (\text{tangent chord theorem}) \\ h + \text{20}\text{°} &= 4h - \text{70}\text{°} & \\ \text{90}\text{°} &= 3h & \\ \therefore h &= \text{30}\text{°} & \end{array}\] \[\begin{array}{rll} P\hat{Q}R &= Q\hat{S}R & (\text{tangent chord theorem}) \\ s &= 4h & \\ &= 4(\text{30}\text{°}) & \\ &= \text{120}\text{°} & \end{array}\] Exercise 8.6 \[\begin{array}{rll} a &= \text{33}\text{°} & (\text{tangent-chord }) \\ b &= \text{33}\text{°} & (\text{alt. angles, } OP \parallel SR) \end{array}\] \[\begin{array}{rll} c &= \text{72}\text{°} & (\text{tangent-chord}) \\ d &= \dfrac{\text{180}\text{°} - \text{72}\text{°} }{2} & (\text{isosceles triangle}) \\ &= \text{54}\text{°} & (\text{alt. angles, } OP \parallel SR) \end{array}\] \[\begin{array}{rll} f &= \text{38}\text{°} & (\text{tangent-chord}) \\ g &= \text{47}\text{°} & (\text{tangent-chord}) \end{array}\] \[\begin{array}{rll} \hat{O}_1 = \hat{Q}_1 &= \text{66}\text{°} & (\text{isosceles, tangent-chord}) \\ \therefore l &= \text{180}\text{°} - 2 \times \text{66}\text{°} & (\text{angles sum } \triangle) \\ &= \text{48}\text{°} & \end{array}\] \[\begin{array}{rll} i &= \text{180}\text{°} - \text{101}\text{°} - \text{39}\text{°} & (\angle \text{s on str. line }) \\ \therefore i &= \text{40}\text{°} & \\ j &= \text{101}\text{°} & (\text{ tangent-chord}) \\ k = i &= \text{40}\text{°} & (\text{ tangent-chord}) \end{array}\] \[\begin{array}{rll} n &= \text{34}\text{°} & (\text{ tangent-chord}) \\ o &= \text{180}\text{°} - \text{90}\text{°} - \text{34}\text{°} & (\text{angles sum } \triangle) \\ \therefore o &= \text{56}\text{°} & \\ m &= \text{56}\text{°} & (\text{ tangent-chord}) \end{array}\] \[\begin{array}{rll} q &= \text{52}\text{°} & (\text{ tangent-chord}) \\ p &= \text{90}\text{°} - \text{52}\text{°} & (\text{tangent perp. radius }) \\ \therefore p &= \text{38}\text{°} & \\ r &= \text{90}\text{°} & (\angle \text{ in semi-circle}) \end{array}\] \(O\) is the centre of the circle and \(SPT\) is a tangent, with \(OP \perp ST\). Determine \(a\), \(b\) and \(c\), giving reasons. \[\begin{array}{rll} a &= \text{90}\text{°} - \text{64}\text{°} & (\text{tangent perp. radius}) \\ &= \text{26}\text{°} & \\ b &= \text{64}\text{°} & (\text{tangent chord}) \\ c &= 2 \times \text{64}\text{°} & (\angle\text{ at centre} = 2\angle \text{ at circum.}) \\ &= \text{128}\text{°} & \\ \end{array}\] \(PAL\) is a tangent to the circle \(ABC\). \[\begin{array}{rll} \hat{A}_1 &= A\hat{C}B & (\text{alt. angles}, AP \parallel BC) \\ A\hat{C}B &= A\hat{B}C & (\angle \text{s opp. equal sides }, AB = AC) \\ \therefore \hat{A}_1 &= A\hat{B}C & \\ \text{Therefore } PAL & \text{is a tangent to circle } ABC & (\angle\text{ between line chord} = \angle\text{ in alt. seg.}) \end{array}\] \(AB\) is a tangent to the circle \(ADP\). \[\begin{array}{rll} \hat{A}_2 &= \hat{B}_2 & (\text{ given }) \\ \text{And } A\hat{P}B &= \hat{B}_2 & (\text{alt. angles}, AP \parallel BC) \\ \text{Therefore } A\hat{P}B &= \hat{A}_2 ABC & \\ \text{Therefore } AB & \text{is a tangent to circle } ADP & (\angle\text{ between line chord} = \angle\text{ in alt. seg.}) \end{array}\] Converse: tangent-chord theorem If a line drawn through the end point of a chord forms an angle equal to the angle subtended by the chord in the alternate segment, then the line is a tangent to the circle. (Reason: \(\angle\) between line and chord \(= \angle\) in alt. seg.) \(BD\) is a tangent to the circle with centre \(O\), with \(BO \perp AD\). Prove that: \(CFOE\) is a cyclic quadrilateral \(FB=BC\) \(\angle A\hat{O}C = 2 B\hat{F}C\) Will \(DC\) be a tangent to the circle passing through \(C,F,O\) and \(E\)? Motivate your answer. \[\begin{array}{rll} BO & \perp OD & (\text{given}) \\ \therefore F\hat{O}E &= \text{90}\text{°} & \\ F\hat{C}E &= \text{90}\text{°} & (\angle \text{ in semi circle}) \\ \therefore CFOE & \text{is a cyclic quad.} & (\text{opp. } \angle \text{s suppl.}) \end{array}\] To show that \(FB = BC\) we first prove \(\triangle BFC\) is an isosceles triangle by showing that \(B\hat{F}C = B\hat{C}F\). \[\begin{array}{rll} B\hat{C}F &= C\hat{E}O & (\text{tangent-chord}) \\ C\hat{E}O &= B\hat{F}C & (\text{ext. } \angle \text{ cyclic quad. } CFOE) \\ \therefore B\hat{F}C &= B\hat{C}F \\ \therefore FB &= BC & (\triangle BFC \text{ isosceles}) \end{array}\] \[\begin{array}{rll} A\hat{O}C &= 2 A\hat{E}C & (\angle \text{ at centre } = 2 \angle \text{ at circum.}) \\ \text{and } A\hat{E}C &= B\hat{F}C & (\text{ext. } \angle \text{ cyclic quad. } CFOE) \\ \therefore A\hat{O}C &= 2 B\hat{F}C \end{array}\] Proof by contradiction. Let us assume that \(DC\) is a tangent to the circle passing through the points \(C\), \(F\), \(O\) and \(E\): \[\begin{array}{rll} \therefore D\hat{C}E = C\hat{O}E \quad (\text{tangent-chord}) \end{array}\] And using the circle with centre \(O\) and tangent \(BD\) we have that: \[\begin{array}{rll} D\hat{C}E &= C\hat{A}E & (\text{tangent-chord}) \\ \text{but } C\hat{A}E &= \frac{1}{2} C\hat{O}E & (\angle \text{ at centre} = 2 \angle \text{ at circum.}) \\ \therefore D\hat{C}E &e C\hat{O}E & \end{array}\] Therefore our assumption is not correct and we can conclude that \(DC\) is not a tangent to the circle passing through the points \(C\), \(F\), \(O\) and \(E\). \(FD\) is drawn parallel to the tangent \(CB\) Prove that: \(FADE\) is a cyclic quadrilateral \(F\hat{E}A = \hat{B}\) \[\begin{array}{rll} F\hat{D}C &= D\hat{C}B & (\text{alt. } \angle\text{s } FD \parallel CB) \\ \text{and } D\hat{C}B &= C\hat{A}E & (\text{tangent-chord}) \\ \therefore F\hat{D}C &= C\hat{A}E \\ \therefore FADE & \text{is a cyclic quad.} & (\angle \text{s in same seg.}) \end{array}\] \[\begin{array}{rll} F\hat{D}A &= \hat{B} & (\text{corresp. } \angle\text{s } FD \parallel CB) \\ \text{and } F\hat{E}A &= F\hat{D}A & (\angle \text{s same seg.\@ cyclic quad.\@ } FADE) \\ \therefore F\hat{E}A &= \hat{B} \end{array}\]